Exercise 1 Let $f(x,y) = \begin{cases} (x^2+y^2)\sin(\frac{1}{x^2+y^2}) & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

a) Show that f is continuous at $(0,0)$

b) Find $\frac{\partial f}{\partial x}(0,0)$, and $\frac{\partial f}{\partial y}(0,0)$

c) Show that f is differentiable at $(0,0)$

Exercise 2 Consider a function $f(x,y)$ such that

$$f(1,2) = 10\ , \qquad \vec{\nabla} f(1,2) = 3\vec{i} + 4\vec{j}\ , \qquad \vec{\nabla} f(3,4) = 5\vec{i} + 6\vec{j}\ , \qquad \vec{\nabla} f(5,6) = \vec{i} + 2\vec{j}$$

a) Find the directional derivative of f at the point $P_0(5,6)$ in the direction of the vector $\vec{v} = (3,4)$

b) Find an approximate value for $f(1.02, 1.99)$

(you can use either methods: 1) $\Delta f = f_x\,\Delta x + f_y\,\Delta y + \varepsilon\sqrt{x^2+y^2}$ *or 2)* $df = (D_u f) \times ds$*)*

c) Find the partial derivative $\frac{\partial}{\partial s}\left[f(s^2+t^2, 3st)\right]_{(s,t)=(1,2)}$

Exercise 3 Suppose that the derivative of the function $f(x,y,z)$ at the point $(1,1,1)$ is greatest in the direction of $\vec{p} = 6\vec{i} - 3\vec{j} + 3\vec{k}$, and that in this direction the value of the derivative is $\sqrt{6}$. Also suppose that

$$f(3,0,-1) = 1\ , \quad \vec{\nabla} f(3,0,-1) = 3\vec{i} - \vec{j} + 5\vec{k}\ , \quad \vec{\nabla} f(3,2,1) = 6\vec{i} - 2\vec{j} + \vec{k}\ , \quad \vec{\nabla} f(0,-1,1) = \vec{i} + \vec{j} + \vec{k}$$

a) Find the directional derivative of f at the point $(3,2,1)$ in the direction of the vector $\vec{i} + \vec{j} + \sqrt{2}\vec{k}$

b) Find $\vec{\nabla} f(1,1,1)$

c) Is there a unit vector $\vec{q}$ such that $D_u f(3,0,-1) = 6$? justify your answer

d) Find the normal line to the surface $f(x,y,z)$ at the point $(3,0,-1)$

e) Let $x = u$, $y = v - 2$, and $z = v - u$, and $g = f(x,y,z)$.
Find $\frac{\partial g}{\partial u}$ and $\frac{\partial g}{\partial v}$ at the point $(u,v) = (3,2)$

f) Let $g = g(u,v)$ be as in part e). Find a plane tangent to the surface

$$g(u,v) = 2w^2 - 1$$

in the uvw-space

(hint: start by finding a point $(u,v,w) = (?,?,?)$ *on the surface)*